

The c -term of the TM 3-body Force: to be or not to be*

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Abstract

In Faddeev calculations of ^3H we study the dependence of the binding energy on the three nucleon force. We adopt the 2π -exchange Tucson-Melbourne three-nucleon force and investigate phenomenologically the dependence on the strength of the individual three-body force operators (a -, b -, c - and d -terms). While the a -term provides a tiny contribution the b - and d -terms are important to gain the experimental binding energy. We find two solutions for the c -term, one around the value used in the Tucson-Melbourne model and a new one close to zero, which supports the recent recommendation of chiral perturbation theory that the short-range c -term should be dropped.

*Dedicated to Prof. Dr. Walter Glöckle on the occasion of his birthday

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I. INTRODUCTION

It is one of the great dreams in the field of few-nucleon physics to find a quantitative correct and theoretically reasonable three nucleon force(3NF). In the past many 3NF models were developed [2–8]. An especially prominent one is the meson-theoretical 3NF, for instance in the form of the Tucson-Melbourne model (TM) [3,4]. The reason for studying 3NFs is the existence of disagreements between the 3N data and the theoretical predictions with NN forces only. First of all the theoretical binding energy of ^3H lacks about 500-800keV in relation to the experimental value of 8.48MeV using recent realistic potentials (e.g. CD-Bonn [9], AV18 [10], Nijmegen 93, Nijmegen I, II [11]). These potentials describe all 2N observables to a degree of accuracy of $\chi^2/N_{data} \sim 1$. In the low energy three nucleon continuum we have demonstrated [1] that most of the observables agree well with the data using nucleon-nucleon (NN) forces only, however, there are exceptions. Some of them are well known as the “ A_y puzzle” [12–16]. In the high energy region the theoretical predictions differ visibly from the data if one only takes NN forces into account. The “Sagara discrepancy” [17–21] is an example of this. These problems definitely require a new Hamiltonian in the realm of the three nucleon system. Moreover, the A_y -puzzle requires not only a 2π -exchange 3NF to become explained [1,16] but other 3NF mechanisms as well.

Beside these low-energy discrepancies in the 3N continuum there are also discrepancies at higher energies. This can be expected naively due to the shorter range nature of the 3NF in comparison to the NN force. Recently it became possible to explain discrepancies between experiment and predictions using NN forces only for the neutron-deuteron (nd) total cross section [22] and the nd elastic differential cross section [23] with the 2π -exchange TM 3NF.

From the point of view of chiral perturbation, this special category of the TM 3NF should be modified [29]. Chiral perturbation theory has been successfully applied in the πN system [24,25] and it is already playing an important role in its application to the NN system as well [26–28]. In [29] it recommended that the pion range - short range part of the c -term of the TM 2π -exchange 3NF should be dropped, based on arguments from chiral perturbation

theory. In doing this the pion range - short range part of the c -term remains, which is of the same type than the a -term. This leads to a redefinition of a . The such modified TM 3NF, called TM', has essentially the same effects on continuum [30] than the original TM 3NF. Much remains to be investigated in the relation between NN and 3NF's.

In the next section we present calculations for the triton binding energy based on variations of the values of the strength parameters in the TM 3NF, individually and combined. The summary and the outlook are give in Section 3.

II. VARIATIONS OF THE TUCSON-MELBOURNE 3NF AND THEIR TRITON BINDING ENERGIES

The TM force has the operator form:

$$V_{TM}^{(3)} = \frac{1}{(2\pi)^6} \frac{g_{\pi NN}^2}{4m^2} \frac{F_{\pi NN}^2(\vec{q}^2) F_{\pi NN}^2(\vec{q}'^2) \vec{\sigma}_1 \cdot \vec{q} \vec{\sigma}_2 \cdot \vec{q}'}{(\vec{q}^2 + m_\pi^2)(\vec{q}'^2 + m_\pi^2)} [O^{\alpha\beta} \tau_\alpha \tau_\beta], \quad (1)$$

$$O^{\alpha\beta} = \delta^{\alpha\beta} [a + b \vec{q} \cdot \vec{q}' + c(\vec{q}^2 + \vec{q}'^2)] - d(\tau_3^\gamma \epsilon^{\alpha\beta\gamma} \vec{\sigma}_3 \cdot \vec{q} \times \vec{q}'). \quad (2)$$

where m_π , m , $g_{\pi NN}$ and $F_{\pi NN}(\vec{q}^2)$ are the pion mass, the nucleon mass, the πNN coupling constant and the vertex function, respectively. The superscript (3) denotes that this expression is only one of three cyclically permuted parts of the total TM 3NF. There are four parameters (a , b , c and d) which are chosen according to certain physical concepts [3,4]. For practical calculations one needs to introduce the vertex function which is normally chosen as

$$F_{\pi NN}(\vec{q}^2) = \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 + \vec{q}^2} \quad (3)$$

The triton binding energy turns out to be strongly dependent on the cut-off parameter Λ . In a phenomenological approach it can be used as a fit parameter to adjust the triton binding energy and this separately for each NN potential [31,32]. Using these cut-off parameters we calculated the polarization transfer parameter $K_y^{y'}$ in the three-body continuum. While the

individual pure NN force predictions are different they essentially coincide if those individually adjusted 3NFs were included and that prediction agrees rather well with the data [33]. This is one example out of several where scaling with the triton binding energy exists for 3N continuum observables. In those studies we kept the original TM parameters (a , b , c and d) and only varied the form-factor cut-off parameter Λ .

Now we want to go one step further and study phenomenologically the dependence of the triton binding energy on the individual terms in the TM 3NF operators connected to the a -, b -, c - and d -term. Like in [32] we solve the Faddeev equation rigorously including the 3NF. We choose CD-Bonn as the NN interaction. The original parameters [4] of the TM model are given in Table 2.1.

TABLES

TABLE I. Parameters for the original 3NF.

$a [m_\pi^{-1}]$	$b [m_\pi^{-3}]$	$c [m_\pi^{-3}]$	$d [m_\pi^{-3}]$	$m_\pi [\text{MeV}]$	$m [\text{MeV}]$	$g_{\pi NN}^2$	$\Lambda [m_\pi]$
1.13	-2.58	1.00	-0.753	139.6	938.926	179.7	4.856

The cut-off parameter Λ is not the original one but adjusted to reproduce the triton binding energy together with CD-Bonn. We multiply each one of the parameters a , b , c and d by a factor X ($0 \leq X \leq 1.5$), one after the other:

$$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \longrightarrow \left\{ \begin{array}{l} \begin{pmatrix} aX \\ b \\ c \\ d \end{pmatrix} \\ \begin{pmatrix} a \\ bX \\ c \\ d \end{pmatrix} \\ \begin{pmatrix} a \\ b \\ cX \\ d \end{pmatrix} \\ \begin{pmatrix} a \\ b \\ c \\ dX \end{pmatrix} \end{array} \right. \begin{array}{l} \text{case a} \\ \text{case b} \\ \text{case c} \\ \text{case d} \end{array} \quad (4)$$

and determine the 3N binding energy for these four cases. The results are shown in Fig. 1.

FIGURES

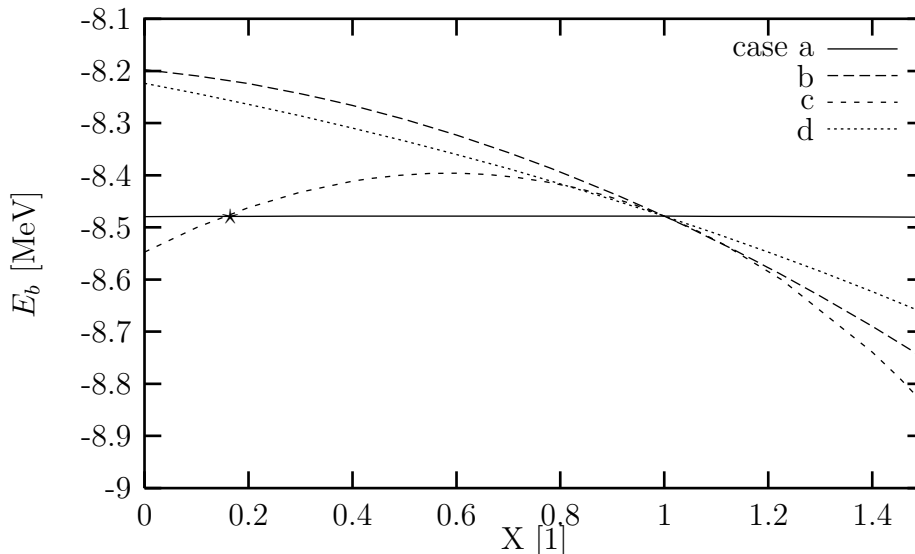


FIG. 1. The binding energy of the triton. The star “★” is a new solution for the c -parameter.

We see that the parameter a contributes negligibly to the 3N bound state and its presence or absence is unimportant. This explains why the prediction for the triton binding energy for the TM and TM' 3NFs are close to each other [30]. The b - and d -terms however are important. The binding energy increases monotonically with their strength. Interestingly, the behaviour of the c -term is such that there are two solutions which lead to the experimental value. We find 0.150 as the new solution (see “★” in Fig.1).

Now, with the exception of a we let all parameters float. The parameter a is kept at its original value 1.13. We search for the sets (b, c, d) which fulfil the condition to produce the experimental binding energy. Thus we have now three independent variables X :

$$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \longrightarrow \begin{pmatrix} a \\ bX_b \\ cX_c \\ dX_d \end{pmatrix} \quad (5)$$

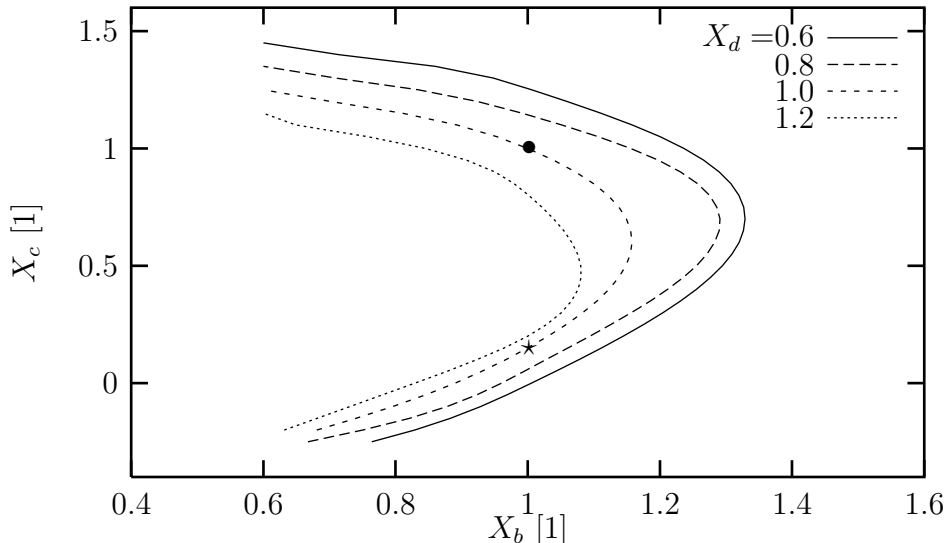


FIG. 2. Contour plot of the parameters b , c , d for constant binding energy.

Fig. 2 shows contour plots for different X_c and X_b while keeping X_d at different fixed values. Each line in Fig. 2 corresponds to the same binding energy (8.48MeV). The black spot indicates the position for the original values of the parameters ($X_b=X_c=X_d=1$). The star is as in Fig. 1. We see that these two solutions for c found above lie on the line in Fig. 2. The b - and c - values to the left (right) of a curve for a particular value of d lead to over-binding (under-binding).

A crucial rule [2] corresponding to the properties of the Δ particle excitation mechanism is that the ratio b/d is 4. The Urbana-Argonne [8] 3NF follows this rule, since except for a phenomenological short range term it includes only the 2π -exchange with an intermediate Δ isobar, the Fujita-Miyazawa 3NF. The TM value for the ratio b/d is 3.43, since in this model the b - and d -term include other processes on top of the 2π -exchange with an intermediate Δ . Also b and d are larger for the TM 3NF than for the pure $\Delta - 2\pi$ -exchange. The values for b and d of the Brazil and RuhrPot 3NF are close to those of the TM 3NF. The Texas 3NF, based on chiral perturbation theory, has even larger values for b and d . This shows that it is not at all clear which values for the strength parameters within the 3NF one should choose.

In Fig. 3 we show the curve for $b=4d$ and of course the additional requirement that the

triton binding energy has the experimental value. As in Fig. 2 the underlying NN potential is CD-Bonn. Of course, in looking to Fig. 3 one should keep in mind that the choice for the $NN\pi$ form-factor (4) leads to a very strong dependence of the strength of the 3NF on the cut-off parameter Λ , as well.

From Table II in [29] the locations of the b and c parameters for several 3NFs are indicated. Note however those 3NFs are not adjusted to the triton binding energy together with the CD-Bonn potential.

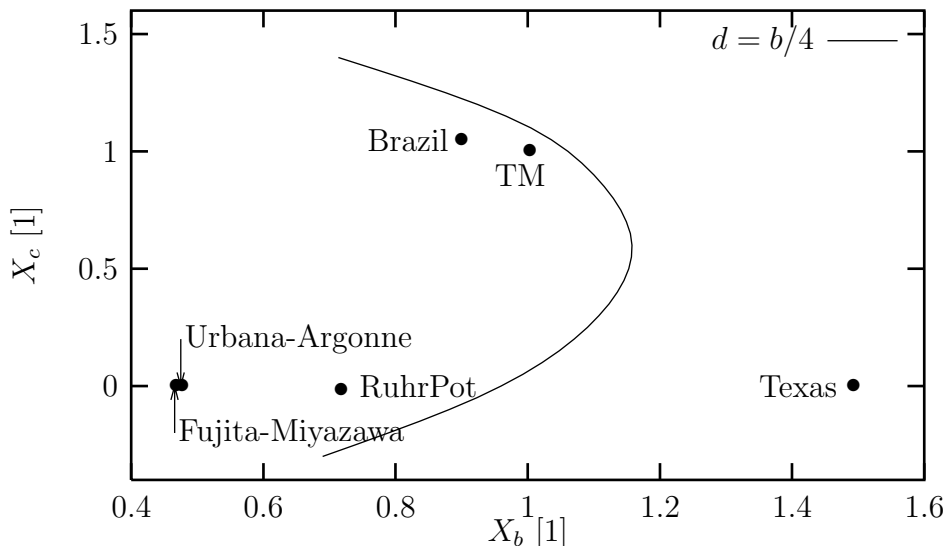


FIG. 3. Parameters for various 3NFs.

III. SUMMARY AND OUTLOOK

Stimulated by [29] we studied the binding energy of ^3H as a function of the strength parameters (a , b , c and d in (2)) in the TM force. We find that the a -term is not decisive when varying in the interval $0 \leq a \leq 2$. The b - and d -terms, however are very important to obtain the experimental value 8.48MeV. Varying c from 0 to $1.5 \times c_{TM}$ we find that there are two solutions which belong to the same binding energy. The new solution is 15% of the original value, namely, $0.15 [m_\pi^{-3}]$. It supports phenomenologically the recommendation given in [29] (based on arguments from chiral perturbation theory) that the short-range

part of the c -term in the TM force should be dropped.

If one assumes a purely phenomenological point of view for choosing the values for a to d in a 3NF of the form (2) Fig. 2 provides a complete overview for the possible values under the requirement that the triton binding energy is gained together with the CD-Bonn potential. Clearly corresponding pictures could be gained for other NN potentials. Of course other 3NF mechanisms have to be explored, too. At least for the A_y -puzzle it is clear that the 2π -exchange 3NF is not sufficient to explain this discrepancy between theory and data. A study of pion range - short range 3NF terms is underway [30] where are predicted by chiral perturbation theory.

Based on the chosen form (2) and the requirement to fit the triton binding energy those 3NFs can now be tested in the 3N continuum with high energy. At IUCF [34], RIKEN [20,35] and KVI measurements are underway for 3N observables between 100-300 MeV. These are cross sections and various spin observables. They will be analysed using the 3NFs fixed in Fig. 2. This might allow to find a preference for a certain region in that parameter space or will show that additional forms are needed.

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